Read the following sentences and fill in the blanks ① to ⑲ with the appropriate formulas. Also, answer the underlined parts (A to C) according to **Questions**.

As shown in Fig. 1, consider a rotation problem of a thin semi-cylindrical shell. The shell is with a radius $R$ and a mass $M$ ignoring its thickness in contact with a smooth horizontal floor and a smooth vertical wall.

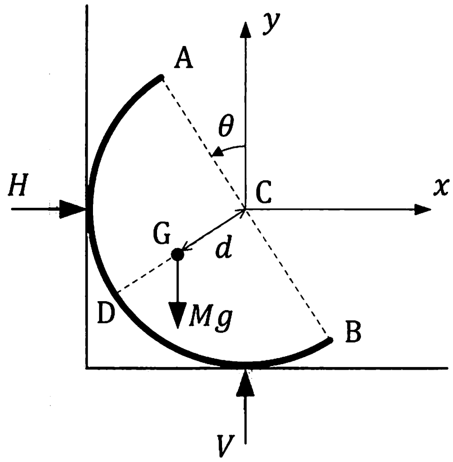


Fig.1

Note that C is the midpoint of the diameter AB and now coincides with the origin O of the *xy*-coordinate system. D is the midpoint of the arc AB, and G is the center of gravity of the shell on the line segment CD. $H$ is the reaction force from the vertical wall, and $V$ is the reaction force from the horizontal floor. The gravitational acceleration is $g$.

Now, a certain force is applied to make the shell stand still at the position where $\theta$ (the angle formed by CA with the y-axis) is equal to 0.(A) After that, when the force is removed instantly, the shell begins to rotate.

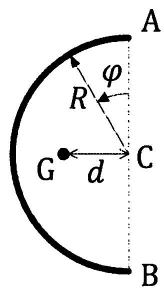


Fig.2

First, as a preparation, let's find the position of the center of gravity G of the shell and its moment of inertia. Denoting the distance between the center of gravity G and the point C by $d$, the following integral equation holds referring to Fig. 2.

$$\int_0^{\pi} \boxed{\quad ① \quad} \sigma R \, d\varphi = 0 \qquad (1)$$

where σ is the mass per unit arc length of the shell $\sigma = \boxed{\quad ② \quad}$.

From Eq.(1), $d = \boxed{\quad ③ \quad}$.

(In the subsequent answers, use $d$.)

On the other hand, the moment of inertia $I_{\mathrm{C}}$ around the point C is obviously $I_{\mathrm{C}} = MR^2$ because all the mass $M$ is on the radius $R$. Therefore, the moment of inertia $I_{\mathrm{G}}$ around the center of gravity G is $I_{\mathrm{G}} = I_{\mathrm{C}} -$ [ ④ ] from the parallel axis theorem of moment of inertia.

If the shell does not move away from the vertical wall, the point C does not move, and then the movement of the shell is only the rotation around the point C. Therefore, the equation of rotational motion is as follows.

$$I_{\mathrm{C}}\ddot{\theta} = \boxed{⑤} \tag{2}$$

Note that the over-dot and the pair of over-dots represent the first and second time derivatives, respectively.

<u>Multiplying both sides of Eq. (2) by $2\dot{\theta}$ and integrating at time $t$, and considering the initial condition of $\dot{\theta} = 0$ at $t = 0$, $\dot{\theta}^2$ can be obtained as follows.</u> (B)

$$\dot{\theta}^2 = \boxed{⑥} \tag{3}$$

This relationship can also be derived from the energy conservation law instead of solving the differential equation (Eq. (2)).

Consider the equation of motion for the center of gravity G in order to obtain the reaction force $H$ from the vertical wall and the reaction force $V$ from the horizontal floor during motion. If the coordinates of the center of gravity G are $(x, y)$, the translational motion can be expressed in the following forms.

$$\left.\begin{aligned} M\ddot{x} &= \\ M\ddot{y} &= \end{aligned}\right. \boxed{⑦} \tag{4}$$

The rotational motion around the center of gravity G can be expressed in the following form.

$$I_{\mathrm{G}}\ddot{\theta} = \boxed{⑧} \tag{5}$$

Since $\ddot{x}$ and $\ddot{y}$ can be expressed as follows in terms of $d$ and $\theta$

$$\ddot{x} = \quad \ddot{y} = \boxed{⑨} \tag{6}$$

the reaction forces $H$ and $V$ can be expressed as follows.

$$H = \quad V = \boxed{⑩} \tag{7}$$

Substituting these equations into the equation of motion around the center of gravity (Eq.(5)) results in the equation of motion relative to the point C (Eq.(2)).

Using Eqs. (2) and (3), the reaction forces can be obtained from Eq. (7) as follows.

$$H = \boxed{⑪} \tag{8}$$

$$V = \boxed{⑫} \tag{9}$$

Since the Eq. (8) of the reaction force $H$ from the vertical wall is negative when $\theta > \pi/2$, the shell will detach from the vertical wall when $\theta = \pi/2$.

In the following, we will consider the movement after detaching from the vertical wall. At the moment of detachment ($t = t_0$), the angle $\theta(t_0)$ and the angular velocity $\dot{\theta}(t_0)$ are as follows.

$$\theta(t_0) = \pi/2 \quad \dot{\theta}(t_0) = \boxed{⑬} \tag{10}$$

<u>The reaction forces are as follows.</u> (C)

$$H(t_0) = 0 \quad V(t_0) = \boxed{⑭} \tag{11}$$

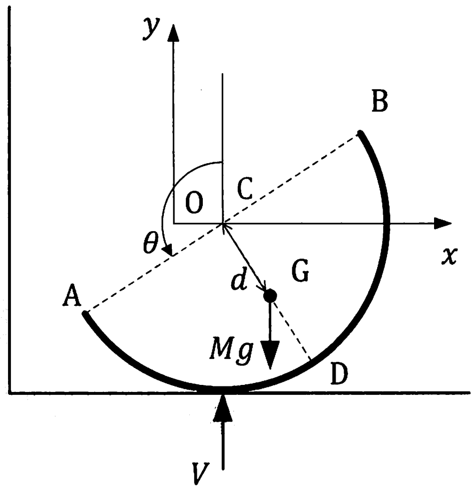

Fig.3

The translational velocity of the center of gravity is as follows.

$$\dot{x}(t_0) = \boxed{⑮} \qquad (12)$$
$$\dot{y}(t_0) = 0$$

Therefore, at the moment of detachment $(t = t_0)$, the distribution of the kinetic energy of the shell can be expressed as the following rates.

$$\text{Rotational motion : Translational motion} = \boxed{⑯} \qquad (13)$$

As shown in Fig. 3, the force acting on the semi-cylindrical shell after detachment is only in the $y$-direction, so the shell continues to move translationally at a constant velocity $\dot{x}(t_0)$ in the $x$-direction while swinging.

The governing equations of motion are derived by setting $H$ to 0 in Eqs. (4) and (5). From those equations, the following equation can be derived.

$$\ddot{\theta} = \frac{1}{2}\frac{d}{d\theta}(\dot{\theta}^2) = \boxed{⑰}\,\dot{\theta}^2 + \boxed{⑱} \qquad (14)$$

This equation is a first-order differential equation for $\dot{\theta}^2$ with respect to $\theta$. Therefore, $\dot{\theta}^2$ can be expressed as a function of $\theta$ by using Eq.(10) as the initial condition. Using the solution, the swing angle when $\dot{\theta}^2 = 0$, that is, the maximum swing angle (maximum value of $\theta$: $\theta_{\max}$) can be obtained. Also, using the energy conservation law, it can be easily obtained as follows.

$$\theta_{\max} = \boxed{⑲} \qquad (\theta_{\max} > \pi/2) \qquad (15)$$

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**Questions**

[1] Determine the required force's magnitude, its acting point and its direction to achieve the state described in the underlined part (A). Also, determine the magnitudes of the reaction forces $H$ and $V$ at that time.

[2] Derive the Eq. (3) using the procedure shown in the underlined part (B).

[3] With respect to the underlined part (C), the reaction force from the horizontal floor $V(t_0)$ is larger than that corresponding to the gravity. Explain the reason of the increment.